# Semantic Cut Elimination for the Logic of Bunched Implications

(as formalized in Coq)

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- BI: a logic for reasoning about (separation of) resources.
- Cut elimination: a proof of  $\vdash \varphi$  only includes subformulas of  $\varphi$ .
- Semantic proof: proof by interpreting syntax in a model.
- Formalized in Coq: axiom-free formalization at

https://github.com/co-dan/BI-cutelim.

BI freely combines intuitionistic and linear connectives:

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$$\mid \mathsf{Emp} \mid \ \varphi \ast \psi \ \mid \ \varphi \rightarrow \psi$$
 
$$\mathsf{Intuitionistic logic}$$

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$$\vdash \mathsf{Linear logic (fragment)}$$

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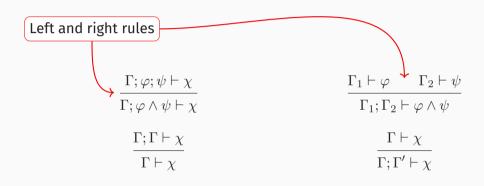
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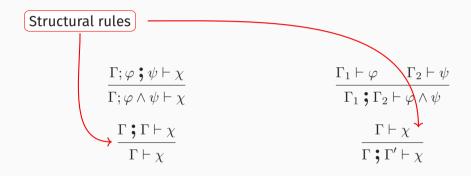
Proposition represent owenership of resources

Sequent: 
$$\Gamma dash \phi$$

$$\frac{\Gamma;\varphi;\psi\vdash\chi}{\Gamma;\varphi\land\psi\vdash\chi}$$

$$\frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1; \Gamma_2 \vdash \varphi \land \psi}$$





$$\frac{\Gamma; \varphi , \psi \vdash \chi}{\Gamma; \varphi * \psi \vdash \chi}$$

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$$\frac{\Gamma \ ; \Gamma \vdash \chi}{\Gamma \vdash \chi}$$

$$\frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1 \cdot \bullet \Gamma_2 \vdash \varphi \land \psi}$$

$$\frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1 \ ; \Gamma_2 \vdash \varphi \land \psi}$$

$$\frac{\Gamma \vdash \chi}{\Gamma \; ; \; \Gamma' \vdash \chi}$$

$$\frac{\Delta(\varphi \bullet \psi) \vdash \chi}{\Delta(\varphi * \psi) \vdash \chi}$$

$$\frac{\Delta(\varphi \; ; \; \psi) \vdash \chi}{\Delta(\varphi \land \psi) \vdash \chi}$$

$$\frac{\Delta(\Gamma \ ; \Gamma) \vdash \chi}{\Delta(\Gamma) \vdash \chi}$$

$$\frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1 \cdot \mathbf{9} \ \Gamma_2 \vdash \varphi \land \psi}$$

$$\frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2 \vdash \psi}{\Gamma_1 \; ; \; \Gamma_2 \vdash \varphi \land \psi}$$

$$\frac{\Delta(\Gamma) \vdash \chi}{\Delta(\Gamma \ ; \Gamma') \vdash \chi}$$

$$\Gamma ::= \varphi \mid \Gamma ; \Gamma \mid \Gamma ; \Gamma \mid \dots$$

# **Cut rule**

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## Intuitions:

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## **Cut rule**

$$\frac{\text{CUT}}{\Delta' \vdash \psi \qquad \Delta(\psi) \vdash \varphi} \frac{\Delta(\Delta') \vdash \varphi}{\Delta(\Delta') \vdash \varphi}$$

## Intuitions:

- $\psi$  is an "intermediate lemma"
- provability relation is transitive

#### **Theorem**

Everything that is provable, is also provable without the cut rule:  $\vdash \varphi \implies \vdash_{\sf cf} \varphi$ 

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# Why eliminate cut?

- makes the calculus analytical (subformula property): any derivation of  $\varphi \vdash \psi$  only involves formula that are already present in  $\varphi$  and  $\psi$
- important ingredient in the automated proof search toolbox

Usually proofs of cut elimination involve analysis by inversion + terminating measure:

$$\frac{\Delta_1; \Delta_2 \vdash \psi_1 \land \psi_2}{\Delta(\Delta_1; \Delta_2) \vdash \varphi} \frac{\Delta(\psi_1 \land \psi_2) \vdash \varphi}{\Delta(\Delta_1; \Delta_2) \vdash \varphi}$$

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For these reason, non-formalized proofs of cut elimination can be fragile and are known to be error-prone.

On the other hand, formalizing these kind of proofs can also be tough...

# Semantic proof of cut elimination

A semantic proof of cut elimination goes through some "universal" model  $\mathcal C$  and the interpretation of sequent calculus in it.

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## **BI** algebra

A BI algebra  $(C, \leq)$  consists of operations  $\top, \bot, \lor, \land, \to$ , Emp, \*, -\* satisfying various laws.

Soundness:  $\varphi \vdash \psi \implies \llbracket \varphi \rrbracket \leq \llbracket \psi \rrbracket$ .

# Intuition: Lindenbaum-Tarski algebra for completeness

$$\text{Define } [\varphi] = \{\psi \mid \varphi \dashv \vdash \psi\} \text{, and } [\varphi] \leq_{\mathcal{L}} [\psi] \iff \varphi \vdash \psi \text{.}$$

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  - Conclusion:  $\varphi \vdash \psi$ .
- The "real" work is to show that  $\mathcal{L}$  is indeed a model.

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Same as cut elimination:  $\varphi \vdash_{\sf cf} \psi \vdash_{\sf cf} \chi \implies \varphi \vdash_{\sf cf} \chi$ 

Attempted solution: use sets of predecessors.

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Solution: close under arbitrary intersections:

$$\mathcal{C} = \{ \bigcap_{i \in I} \langle \varphi_i \rangle \mid I \text{ arbitrary set}, \varphi_i \in \mathit{Frml} \} \subseteq \wp(\mathit{Bunch})$$

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$$\begin{aligned} \operatorname{cl}(-) &: \wp(\mathit{Bunch}) \to \mathcal{C} \\ \operatorname{cl}(X) &= \bigcap \{ \langle \varphi \rangle \mid X \subseteq \langle \varphi \rangle \} \end{aligned}$$

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The smallest set in  $\mathcal C$  containing X

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Lift operations to C:

$$X \wedge Y = X \cap Y$$

$$X\vee Y=\operatorname{cl}(X\cup Y)$$

$$X*Y=\operatorname{cl}(\{\Delta_1\ {}_{\!\raisebox{1pt}{\text{\circle*{1.5}}}}\Delta_2\mid \Delta_1\in X, \Delta_2\in Y\})$$

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- More modular proof
- Extensions: structural rules,  $\square$  modality.

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 $\bullet$  Good representation for  ${\cal C}$  makes life easier

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Record C := {
  CPred :> Bunch → Prop;
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- Setoids and setoid rewriting were helpful, useful type classes in stdpp
- Turn equations  $\Delta = \Delta'(\Gamma)$  into inductive systems  ${\tt Inductive \ bunch\_decomp : bunch \to bunch\_ctx \to bunch \to Prop}$

## Thank you

Thank you for listening!

Let me know if you have questions, d.frumin@rug.nl.